Gluon emission in interaction of two reggeons

M.A.Braun, S.S.Pozdnyakov, M.Yu.Salykin, M.I.Vyazovsky
S.Petersburg State University, Russia

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Abstract

The vertex is constructed for gluon production in interaction of two reggeons coupled to projectiles and two reggeons coupled to targets. The vertex can be used to build cross-sections for collisions of two pairs of nucleons in AA scattering. Transversality of the constructed vertex is demonstrated as well as its good behaviour at large longitudinal momenta necessary for applications. Poles at zero values of longitudinal momenta are discussed and it is found that they remain in the amplitudes unlike in the case of a single projectile.

1 Introduction

In the QCD hadronic interactions at high energies in the Regge kinematics, when the transferred transverse momenta are much smaller than energies, can be described by the interaction of normal gluons with reggeized ones ("reggeons"). The latter combine into pomerons coupled to participant colourless projectiles and targets. The simplest case is the collision of the highly virtual gluon with a hadron or a nucleus. It has been studied long ago and solved by formulation of the BFKL [1, 2] and Balitski-Kovchegov [3, 4] equations for the non-integrated gluon densities, which allow to calculate the relevant cross-sections. With some ingenuity this approach can be generalized to pp or pA scattering. However, nucleus-nucleus collisions present a more difficult problem. In the case of heavy nuclei the total cross-sections can be treated within the effective pomeron interaction formalism [5]. For the inclusive gluon production a general formalism was developed in [6, 7, 8] in the framework of the Color Glass Condensate (CGC) approach. The inclusive cross-sections were expressed via averages of the gluon potentials in the field of the colliding nuclei, developed in rapidity according to the so-called JIMWLK functional equations. These averages can only be found by numerical methods. Several attempts to find analytic expressions for the inclusive gluon production have lead to only approximate [9] or partial and inconclusive results [10, 11].

The BFKL approach presents an alternative (in all probability equivalent, in principle) way to study this problem. It may produce analytic formulas for the cross-sections and also allow to study the case of light nuclei, when the leading contributions appear to be of the subleading order in N_c [12] and a direct application of the CGC approach does not seem to be possible. In the BFKL approach the problem requires knowledge of amplitudes for gluon production in interaction of many reggeons coupled to the projectile with many reggeons coupled to the target.

The simplest non-trivial case is production of gluons in interaction of two reggeons coupled to the projectile with two reggeons coupled to the target. For heavy nuclei, in the lowest order, this means interaction of two nucleons from each of the nuclei, as illustrated in Fig. 1. One observes that apart from the well-known production amplitude in collision between two colourless objects there enters a more complicated vertex when the gluon is produced by

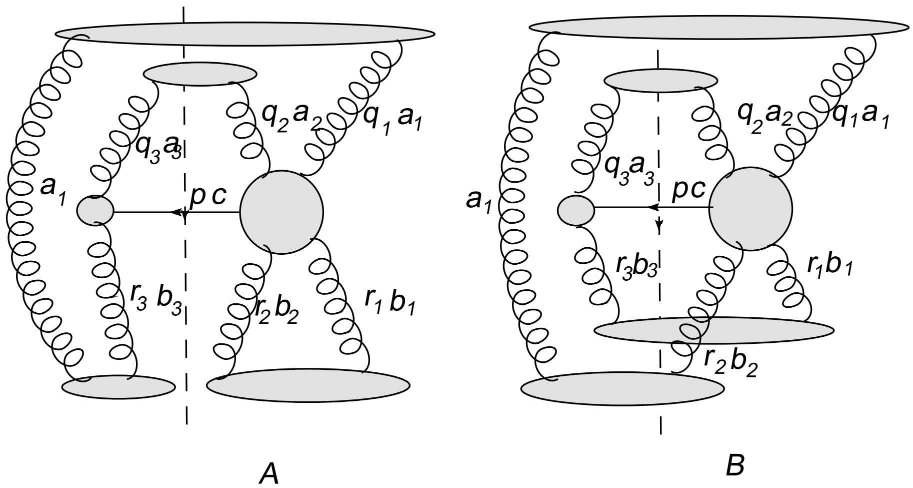

Figure 1: Gluon production by pair of nucleons of the projectile in collision with a pair of nucleons from the target. Solid lines correspond to particles, wavy ones to reggeons

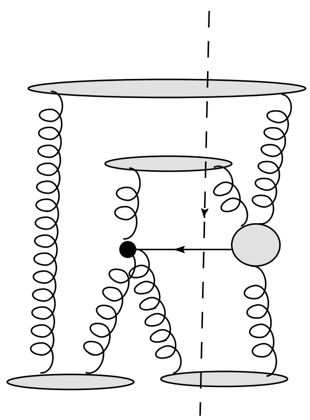

Figure 2: Gluon production by pair of nucleons of the projectile in collision with a pair of nucleons from the target from cutting the RR→RRP vertex. The left vertex is induced.

a pair of reggeons coupled to projectile and target nucleons (the RR→RRP vertex where P stands for "particle", that is gluon).

In this paper we study this RR→RRP vertex both with a real (on-mass-shell) and virtual (off-mass-shell) gluon. The latter is needed if one wants to construct the vertex for transition of three reggeons into three reggeons (RRR→RRR), which enters the kernel of the equation for the odderon or the higher pomeron made of the three reggeons. Note that for the odderon the RRR→RRR vertex, integrated over the longitudinal momenta, was derived in [13]

Note that for collisions of heavy nuclei the contribution shown in Fig. 1 is only a particular term corresponding to interaction of only two pairs of nucleons. By itself is corresponds only gluon production in collision of two deuterons. But even in this simple case one should additionally consider contributions from cutting the vertex itself, as shown in Fig. 2. As seen from Fig. 1, the immediate application of the vertex RR→RRP is to give a non-trivial contribution to the diffractive gluon production in deuteron-proton collisions (Fig. 1,A)

In this paper we do not attempt to calculate this contribution, which requires a lot more of analytical and numerical effort. Our aim is to just derive the vertex itself and demonstrate it basic properties important to is subsequent applications: transversality, vanishing at large longitudinal momenta and presence or absence of poles at zero values of the latter.

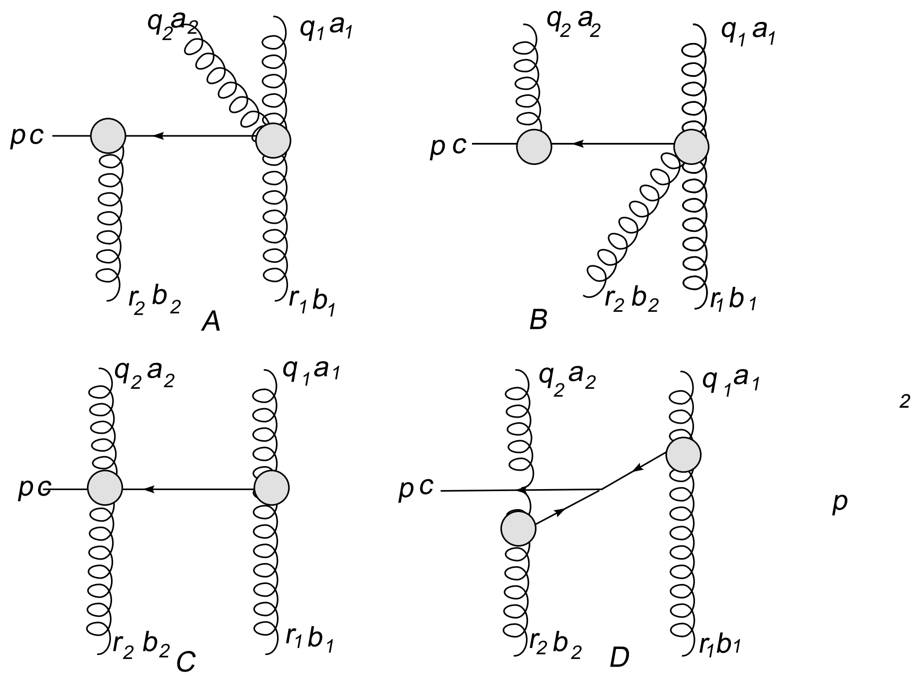

Figure 3: Different contributions to the vertex RR→RRP. Solid lines correspond to particles, wavy ones to reggeons

The main tool for the calculation of the vertex is the Lipatov effective action [14], which gives the rules for reggeon-particle interaction at a given rapidity and introduces the so-called induced vertices for this interaction. Some of the full and induced vertices derived from this action have been already found in [15]. The induced vertex RP→RP is new and will be derived here.

The paper is organized as follows. In the next section we derive the vertex RR→RRP off the mass shell. In Section 3. we demonstrate its transversality. Section 4 is devoted to the study of the vertex at high longitudinal momenta. In Section 5 we derive the vertex on the mass-shell. In Section 6 we investigate the pole singularities at zero values of the longitudinal momenta. In the last section we make some conclusions.

2 Vertex RR→RRP with a virtual emitted gluon

In the framework of the effective action the vertex RR→RRP is constructed as a sum of four diagrams, shown in Fig.3, A,...,D with subsequent symmetrization in the reggeons attached to the projectile (upper in Fig. 3) and those attached to the target (lower in Fig. 3). The blobs in Fig. 3 denote full (basic plus induced) vertices. Solid lines denote gluons, wavy ones correspond to reggeons. We denote momenta and colours of upper reggeons from right to left as q_1, a_1 and q_2, a_2 and those for lower reggeons as r_1, b_1 and r_2, b_2. The emitted gluon has its momentum, polarization and colour p, μ, c. The reggeons carry polarization vectors $n^{\pm}$ with $n^+_+ = n^+_\perp = n^-_- = n^-_\perp = 0$, $n^+_- = n^-_+ = 1$.

2.1 Fig. 3,A

As is clear from the figure we already know all the building blocks for the construction of the vertex. The vertex on the right RR→RP can be found from the vertex R→RRP, calculated

in [15] after changing the direction of reggeon propagation and notations of the momenta. From the same publication one can extract the vertex on the left P→RP.

In this way we find the vertex RR→RP on the right in the form

$$\bar{V}_\nu = i\frac{f^{b_1a_1e}f^{ea_2d}}{t_1^2+i0}\Big(\bar{A}t_\nu - \bar{B}q_{1\nu} - \bar{C}q_{2\nu} + \bar{D}n^+_\nu + \bar{E}n^-_\nu\Big), \tag{1}$$

where $t = p + r_2 = q_1 + q_2 - r_1$, $t_1 = q_1 - r_1$ and

$$\bar{A} = 3t_- + \frac{r_1^2}{q_{1+}}, \quad \bar{B} = 4t_-, \quad \bar{C} = 4t_- + 2\frac{r_1^2}{q_{1+}},$$

$$\bar{D} = -\frac{r_1^2(r_1-q_1)^2}{t_+q_{1+}} - 2t_-\frac{r_1^2}{q_{1+}} - 4t_-^2,$$

$$\bar{E} = -\Big(-(r_1+q_1)(t+q_2)+q_2^2-q_1^2+(r_1-q_1)^2+2r_{1-}q_{1+}\Big)+\Big(-2r_{1-}+\frac{r_1^2}{q_{1+}}\Big)\Big(t_++q_{2+}-\frac{q_2^2}{r_{1-}}\Big). \tag{2}$$

Note that this vertex is transversal $(t\bar{V}) = 0$.

On the left of the diagram Fig. 3,A there stands the vertex R→RP, given by the tensor [15]

$$X_{\mu\nu} = -gf^{db_2c}\Big((p+t)_+g_{\mu\nu} + (p-2t)_\mu n^+_\nu + (t-2p)_\nu n^+_\mu - n^+_\mu n^+_\nu\frac{r_2^2}{p_+}\Big). \tag{3}$$

This vertex is not orthogonal to p or t separately. However, the product $(pXt) = 0$. The term $t_\nu n^+_\mu$ does not contribute due to the transversality of $\bar{V}_\nu$ and will be dropped.

Multiplying X by $\bar{V}$ on the right we obtain the contribution to the vertex RR→RRP from the diagram in Fig. 3,A:

$$\mathcal{A}_{1\mu} = -g^3C_1\frac{1}{(t_1^2+i0)(t^2+i0)}\Big(a_\mu\bar{A} - b_\mu\bar{B} - c_\mu\bar{C} + d_\mu\bar{D} + e_\mu\bar{E}\Big), \tag{4}$$

where vectors $a, ...e$ are

$$a_\mu = p_\mu p_+ - n^+_\mu(t^2+p^2),$$

$$b_\mu = 2p_+q_{1\mu} + (p-2t)_\mu q_{1+} - n^+_\mu\Big(2(pq_1) + r_2^2\frac{q_{1+}}{p_+}\Big),$$

$$c_\mu = 2p_+q_{2\mu} + (p-2t)_\mu q_{2+} - n^+_\mu\Big(2(pq_2) + r_2^2\frac{q_{2+}}{p_+}\Big),$$

$$d_\mu = 0,$$

$$e_\mu = 2p_+n^-_\mu + (p-2t)_\mu - n^+_\mu\Big(2p_- + \frac{r_2^2}{p_+}\Big)$$

and the colour coefficient C_1 is

$$C_1 = f^{db_2c}f^{b_1a_1e}f^{ea_2d}. \tag{5}$$

2.2 Fig. 3,B

In this case the R→RRP vertex on the right can be taken directly from [15] duly changing the notations. We get for it

$$V_\nu = i\frac{f^{db_2e}f^{eb_1a_1}}{t_1^2+i0}\Big\{q_{1+}(4r_1+\bar{t})_\nu - \Big[(q_1+r_1)(\bar{t}-r_2) + r_2^2 - r_1^2 + (q_1-r_1)^2 + 2q_{1+}r_{1-}\Big]n^+_\nu$$

$$+\frac{q_1^2(q_1-r_1)^2}{\bar{t}_-r_{1-}}n^-_\nu+\Big(2q_{1+}-\frac{q_1^2}{r_{1-}}\Big)\Big[-2q_{1+}n^-_\nu+(\bar{t}+2r_2)_\nu+\Big(\bar{t}_--r_{2-}+\frac{r_2^2}{q_{1+}}\Big)n^+_\nu\Big]\Big\},$$

where $\bar{t}=q_1-r_1-r_2$. We rewrite it as

$$V_\nu=i\frac{f^{db_2e}f^{eb_1a}}{(q_1-r_1)^2+i0}\Big\{A\bar{t}_\nu+Br_{1\nu}+Cr_{2\nu}+Dn^-_\nu+En^+_\nu\Big\},$$

where

$$A=3q_{1+}-\frac{q_1^2}{r_{1-}},\quad B=4q_{1+},\quad C=2\Big(2q_{1+}-\frac{q_1^2}{r_{1-}}\Big),$$

$$D=\frac{q_1^2(q_1-r_1)^2}{\bar{t}_-r_{1-}}-2q_{1+}\Big(2q_{1+}-\frac{q_1^2}{r_{1-}}\Big),$$

$$E=-\Big((q_1+r_1)(\bar{t}-r_2)+r_2^2-r_1^2+(q_1-r_1)^2+2q_{1+}r_{1-}\Big)+\Big(2q_{1+}-\frac{q_1^2}{r_{1-}}\Big)\Big(\bar{t}_--r_{2-}+\frac{r_2^2}{q_{1+}}\Big). \quad (6)$$

Vertex $\bar{X}$ in the diagram on the left is obtained from (3) by inversion of reggeons with p and $\bar{t}$ preserved:

$$\bar{X}_{\mu\nu}=-gf^{da_2c}\Big(2p_-g_{\mu\nu}+(p-2\bar{t})_\mu n^-_\nu+n^-_\mu(\bar{t}-2p)_\nu-n^-_\mu n^-_\nu\frac{q_2^2}{p_-}\Big). \quad (7)$$

Multiplying it by V on the right we get the contribution to the vertex RR→RRP from the diagram in Fig. 3,B:

$$\mathcal{A}_{2\mu}=-g^3C_2\frac{1}{(t_1^2+i0)(\bar{t}^2+i0)}\Big(\bar{a}_\mu A+\bar{b}_\mu B+\bar{c}_\mu C+\bar{d}_\mu D+\bar{e}_\mu E\Big), \quad (8)$$

where vectors $\bar{a}, ...\bar{e}$ are

$$\bar{a}_\mu=p_\mu p_--n^-_\mu(\bar{t}^2+p^2),$$

$$\bar{b}_\mu=2p_-r_{1\mu}+(p-2\bar{t})_\mu r_{1-}-n^-_\mu\Big(2(pr_1)+q_2^2\frac{r_{1-}}{p_-}\Big),$$

$$\bar{c}_\mu=2p_-r_{2\mu}+(p-2\bar{t})_\mu r_{2-}-n^-_\mu\Big(2(pr_2)+q_2^2\frac{r_{2-}}{p_-}\Big),$$

$$\bar{d}_\mu=0,$$

$$\bar{e}_\mu=2p_-n^+_\mu+(p-2\bar{t})_\mu-n^-_\mu\Big(2p_++\frac{q_2^2}{p_-}\Big)$$

and the colour factor is

$$C_2=f^{da_2c}f^{db_2e}f^{eb_1a_1}\ . \quad (9)$$

2.3 Fig. 3,C

Here on the right we have the well-known Lipatov vertex $f^{a_1b_1d}L_{1\nu}$, where the momentum part is

$$L_{1\nu}=a_{1\nu}+b_1n^+_\nu+c_1n^-_\nu. \quad (10)$$

Here

$$a_1=q_1+r_1,\quad b_1=\frac{r_1^2}{q_{1+}}-2r_{1-},\quad c_1=\frac{q_1^2}{r_{1-}}-2q_{1+}.$$

On the left, however, we have a new vertex RP→RP. Using the effective action we find that it consists of two terms, the term coming from the standard 4-gluon interaction Z_1 and the induced term Z_2. Calculations give

$$Z^{(1)}_{\mu\nu}=ig^2\Big[f^{a_2ce}f^{b_2de}\Big(2n^+_\mu n^-_\nu-n^-_\mu n^+_\nu-g_{\mu\nu}\Big)+f^{a_2de}f^{b_2ce}\Big(2n^-_\mu n^+_\nu-n^+_\mu n^-_\nu-g_{\mu\nu}\Big)\Big] \quad (11)$$

and

$$Z^{(2)}_{\mu\nu} = ig^2 q^2_{2\perp}\left(\frac{f^{a_2ce}f^{b_2de}}{-p_-r_{2-}} + \frac{f^{a_2de}f^{b_2ce}}{t_{1-}r_{2-}}\right) n^-_\mu n^-_\nu + ig^2 r^2_{2\perp}\left(\frac{f^{a_2ce}f^{b_2de}}{-t_{1+}q_{2+}} + \frac{f^{a_2de}f^{b_2ce}}{p_+q_{2+}}\right) n^+_\mu n^+_\nu \ . \tag{12}$$

Multiplying these terms by the Lipatov vertex from the right we correspondingly get two terms

$$\mathcal{B}^{(1)}_\mu = g^3 f^{a_1b_1d}\frac{1}{t_1^2+i0}\Big\{f^{a_2ce}f^{b_2de}\Big[2a_{1-}n^+_\mu - a_{1+}n^-_\mu - a_{1\mu}$$
$$+n^+_\mu\Big(\frac{r_1^2}{q_{1+}} - 2r_{1-}\Big) - 2n^-_\mu\Big(\frac{q_1^2}{r_{1-}} - 2q_{1+}\Big)\Big]$$
$$+f^{a_2de}f^{b_2ce}\Big[2a_{1+}n^-_\mu - a_{1-}n^+_\mu - a_{1\mu} - 2n^+_\mu\Big(\frac{r_1^2}{q_{1+}} - 2r_{1-}\Big) + n^-_\mu\Big(\frac{q_1^2}{r_{1-}} - 2q_{1+}\Big)\Big]\Big\}$$

and

$$\mathcal{B}^{(2)}_\mu = g^3 f^{a_1b_1d}\frac{1}{t^2+i0}\Big[n^-_\mu z_1\Big(\frac{r_1^2}{q_{1+}} + a_{1-} - 2r_{1-}\Big) + n^+_\mu z_2\Big(\frac{q_1^2}{r_{1-}} + a_{1+} - 2q_{1+}\Big)\Big],$$

where we denoted

$$z_1 \equiv q^2_{2\perp}\left(\frac{f^{a_2ce}f^{b_2de}}{-p_-r_{2-}} + \frac{f^{a_2de}f^{b_2ce}}{t_{1-}r_{2-}}\right)$$

and

$$z_2 \equiv r^2_{2\perp}\left(\frac{f^{a_2ce}f^{b_2de}}{-t_{1+}q_{2+}} + \frac{f^{a_2de}f^{b_2ce}}{p_+q_{2+}}\right).$$

In practice it is convenient to split the total contribution into two parts having different colour factors. So the contribution from Fig. 3,C consists of two different amplitudes

$$\mathcal{A}_{3\mu} = g^3 C_3\frac{1}{t_1^2+i0}\Big[2a_{1-}n^+_\mu - a_{1+}n^-_\mu - a_{1\mu} + n^+_\mu\Big(\frac{r_1^2}{q_{1+}} - 2r_{1-}\Big) - 2n^-_\mu\Big(\frac{q_1^2}{r_{1-}} - 2q_{1+}\Big)$$
$$+ n^-_\mu\frac{q_2^2}{-p_-r_{2-}}\Big(\frac{r_1^2}{q_{1+}} + a_{1-} - 2r_{1-}\Big) + n^+_\mu\frac{r_1^2}{-t_{1+}q_{2+}}\Big(\frac{q_1^2}{r_{1-}} + a_{1+} - 2q_{1+}\Big)\Big] \tag{13}$$

and

$$\mathcal{A}_{4\mu} = g^3 C_4\frac{1}{t_1^2+i0}\Big[2a_{1+}n^-_\mu - a_{1-}n^+_\mu - a_{1\mu} - 2n^+_\mu\Big(\frac{r_1^2}{q_{1+}} - 2r_{1-}\Big) + n^-_\mu\Big(\frac{q_1^2}{r_{1-}} - 2q_{1+}\Big)$$
$$+ n^-_\mu\frac{q_2^2}{t_{1-}r_{2-}}\Big(\frac{r_1^2}{q_{1+}} + a_{1-} - 2r_{1-}\Big) + n^+_\mu\frac{r_1^2}{p_+q_{2+}}\Big(\frac{q_1^2}{r_{1-}} + a_{1+} - 2q_{1+}\Big)\Big], \tag{14}$$

where the colour factors are

$$C_3 = f^{a_1b_1d}f^{a_2ce}f^{b_2de} = -C_2 \tag{15}$$

and

$$C_4 = f^{a_1b_1d}f^{a_2de}f^{b_2ce} = C_1\ . \tag{16}$$

2.4 Fig. 3,D

The contribution from this diagram comes from two Lipatov vertices $L_{1\nu_1}$ and $L_{2\nu_2}$ coupled to the triple gluon vertex

$$\Gamma_{\nu_1\mu,\nu_2}(t_1,p,t_2) = -gf^{d_1cd_2}\Big((p+t_2)_{\nu_1}g_{\mu\nu_2} + (t_1-t_2)_\mu g_{\nu_1\nu_2} + (-t_1-p)_{\nu_2}g_{\mu\nu_1}\Big),$$

where $t_1 = q_1 - r_1$, $t_2 = q_2 - r_2$ The two Lipatov vertices are transversal $(t_1L_1) = (t_2L_2) = 0$. So in $\Gamma_{\nu_1\mu\nu_2}$ we can drop terms with $t_{1\nu_1}$ and $t_{2\nu_2}$ and take

$$\Gamma_{\nu_1\mu\nu_2} = -f^{d_1cd_2}\Big(2t_{2\nu_1}g_{\mu\nu_2} + (t_1-t_2)_\mu g_{\nu_1\nu_2} - 2t_{1\nu_2}g_{\mu_1}\Big). \qquad (17)$$

As a result we find a compact expression for the contribution from Fig. 3,D

$$\mathcal{A}_{5\mu} = g^3C_5\frac{1}{(t_1^2+i0)(t_2^2+i0)}\Big(2(t_2L_1)L_{2\mu} - 2(t_1L_2)L_{1\mu} + (L_1L_2)(t_1-t_2)_\mu\Big). \qquad (18)$$

Here the colour factor is

$$C_5 = f^{a_1d_1b_1}f^{d_1cd_2}f^{a_2d_2b_2} = C_1 + C_2, \qquad (19)$$

where we have used the Jacoby identity.

Note that for some calculations a more explicit form may be preferable, which is given in the Appendix.

2.5 Symmetrization and particular configurations

Contributions calculated above refer to a fixed order of upper and lower reggeons

$$\mathcal{A}(q_2,a_2;q_1,a_1|r_2,b_2;r_1,b_1) \equiv \mathcal{A}(2,1|2,1).$$

The total amplitude is obtained after we sum it with the contributions with interchange of upper and lower gluons For $\mathcal{A}_i$ with $i = 1,2,3,4$ each interchange gives a new diagram so that the total amplitude is

$$\mathcal{A}_i^{tot} = \mathcal{A}_i(2,1|2,1) + \mathcal{A}_i(2,1|1,2) + \mathcal{A}_i(1,2|2,1) + \mathcal{A}_i(1,2|1,2), \quad i = 1,2,3,4.$$

For $i = 5$ simultaneous interchange of upper and lower reggeons does not give a new contribution. So in this case

$$\mathcal{A}_5^{tot} = \mathcal{A}_5(2,1|2,1) + \mathcal{A}_5(2,1|1,2).$$

Each interchange combines interchange of momenta and colours. In the general case this introduces a multitude of different colour factors. To simplify we restrict ourselves with colour configurations actually present in the applications. Inspecting Fig. 1 we see that the RR→RRP vertex may appear in two different colour configurations. One of them, Fig. 1,A is diffractive with respect to the targets but non-diffractive with respect to the projectiles, D-ND configuration (of course there exists a similar configuration with projectiles and targets reversed). The other configuration, that of Fig. 1,B is non-diffractive with respect to both projectiles and targets, the ND-ND configuration. In the D-ND configuration the general colour coefficient $C(a_2,a_1|b_2,b_1)$ is to be convoluted with δ_{b_1,b_2} . Then we obtain

$$C(a_2,a_1|b_2,b_1)\delta_{b_1b_2} = Nf^{a_2a_1c}\kappa^{D-ND}, \qquad (20)$$

where κ is just the number, different for different diagrams. In the ND-ND configuration the general colour coefficient is to be convoluted with $\delta_{a_1b_2}$ and we obtain

$$C(a_2,a_1|b_2,b_1)\delta_{a_1b_2} = Nf^{a_2b_1c}\kappa^{ND-ND}, \qquad (21)$$

where again the number κ is different for different diagrams. The choice of convolution colours a_1 and b_2 is of course arbitrary due to symmetry in both upper and lower reggeons.

Using these considerations for both configurations the total amplitudes can be presented via their momentum parts as follows. In the D-ND configuration

$$\mathcal{A}_i^{tot} = Nf^{a_2a_1c} = \kappa_i^{(1)}\mathcal{A}_i(q_2, q_1|r_2, r_1) + \kappa_i^{(2)}\mathcal{A}_i(q_2, q_1|r_1, r_2)$$
$$+ \kappa_i^{(3)}\mathcal{A}_i(q_1, q_2|r_2, r_1) + \kappa_i^{(4)}\mathcal{A}_i(q_1, q_2|r_1, r_2). \tag{22}$$

In the ND-ND configuration we have the same formula with $f^{a_2a_1c} \to f^{a_2b_1c}$.

Simple calculations give for D-ND configuration

$$\kappa_i^{(1)} = \kappa_i^{(2)} = -\kappa_i^{(3)} = -\kappa_i^{(4)}$$

and

$$\kappa_1^{(1)} = \kappa_4^{(1)} = -\frac{1}{2}, \quad \kappa_2^{(1)} = -\kappa_3^{(1)} = 1, \quad \kappa_5^{(1)} = \frac{1}{2}$$

We recall that for $i = 5$ in (22) only the first two terms are to be taken into account.

For the ND-ND configuration coefficients $\kappa_i^{(k)}$ are given in the table.

Table of $\kappa_i^{(k)}$ for ND-ND configuration

$k =$	1	2	3	4
κ_1	1/2	0	1	1
κ_2	-1	0	-1	-1/2

Other coefficients are defined through κ_1 and κ_2 according to relations

$$\kappa_3 = -\kappa_2, \quad \kappa_4 = \kappa_1, \quad \kappa_5 = \kappa_1 + \kappa_2$$

3 Transversality

3.1 Amplitudes $\mathcal{A}_i$, $i = 1, ...5$

The obtained expressions for RR→ RRP amplitudes are rather cumbersome. A simple method to check transversality is just to calculate the product $(p\mathcal{A})$ numerically. The corresponding calculations by a FORTRAN program in both D-ND and ND-ND configurations show that the constructed vertex is indeed transversal. Nevertheless it is instructive to see this fact analytically.

We consider subsequently the five amplitudes $\mathcal{A}_i$, $i = 1, ...5$ corresponding to contributions (4), (8), (13), (14) and (18) respectively.

1. $\mathcal{A}_1$

Using orthogonality of $\bar{V}$ we find

$$X_1 = (p\mathcal{A}_1) = g^3 C_1 \frac{1}{t_1^2}(\bar{A}t_+ - \bar{B}q_{1+} - \bar{C}q_{2+} + \bar{E}) = g^3 C_1 \frac{1}{t_1^2} Z_1, \tag{23}$$

where $t_1 = q_1 - r_1$ and $\bar{A}, ...\bar{E}$ are given by (2). We find

$$Z_1 = -3r_{1-}(q_{1+} + q_{2+}) + (a_1, t + q_2) + q_1^2 + q_2^2 + 2r_1^2 - t_1^2 + r_1^2 \frac{q_{2+}}{q_{1+}} - \frac{q_2^2 r_1^2}{q_{1+} r_{1-}}, \tag{24}$$

where $a_1 = q_1 + r_1$.

2. $\mathcal{A}_2$

Similarly to (23) we present

$$X_2=(p\mathcal{A}^{(2)})=g^3C_2\frac{1}{t_1^2}(At_-+Br_{1-}+Cr_{2-}+E)=g^3C_2\frac{1}{t_1^2}Z_2. \tag{25}$$

The explicit expressions for $A,...E$ are given in (6) We find

$$Z_2=-3q_{1+}(r_{1-}+r_{2-})-(a_1,\bar{t}-r_2)+r_1^2+r_2^2+2q_1^2-t_1^2+q_1^2\frac{r_{2-}}{r_{1-}}-\frac{q_1^2r_2^2}{q_{1+}r_{1-}} \tag{26}$$

3. $\mathcal{A}_3$

We have

$$p^\mu Z^{(3)}_{\mu\nu}=if^{a_2ce}f^{b_2de}\Big[n^+_\nu\Big(2p_+-\frac{q^2}{r_{2-}}\Big)-n^-_\nu\Big(p_-+\frac{r_2^2p_+}{t_{1+}q_{2+}}\Big)-p_\nu\Big].$$

This has to be multiplied by $L_{1\nu}$ given by (10) We get

$$X_3=(p\mathcal{A}^{(3)})=g^3C_3\frac{1}{t_1^2}Z_3,$$

Calculations give

$$Z_3=3q_{1+}p_--(a_1p)+r_1^2+r_2^2+2q_1^2+2q_1^2\frac{r_{2-}}{r_{1-}}+q_2^2\frac{r_{1-}}{r_{2-}}+r_1^2\frac{q_{2+}}{q_{1+}}+r_2^2\frac{q_{1+}}{q_{2+}}$$
$$-\frac{q_2^2r_1^2}{q_{1+}r_{2-}}-\frac{q_1^2r_2^2}{q_{1+}r_{1-}}-\frac{q_1^2r_2^2}{q_{2+}r_{1-}}. \tag{27}$$

4. $\mathcal{A}_4$

We have

$$p^\mu Z^{(4)}_{\mu\nu}=if^{a_2de}f^{b_cde}\Big[n^+_\nu\Big(2p_-+\frac{r^2}{q_{2+}}\Big)-n^-_\nu\Big(p_+-\frac{q_2^2p_-}{t_{1-}r_{2-}}\Big)-p_\nu\Big].$$

This again has to be multiplied by (10). We get

$$X_4=(p\mathcal{A}^{(4)})=g^3C_4\frac{1}{t_1^2}Z_4,$$

where after simple calculations

$$Z_4=3r_{1-}p_+-(a_1p)-q_1^2-q_2^2-2r_1^2-2r_1^2\frac{q_{2+}}{q_{1+}}-r_2^2\frac{q_{1+}}{q_{2+}}-q_1^2\frac{r_{2-}}{r_{1-}}-q_2^2\frac{r_{1-}}{r_{2-}}$$
$$+\frac{q_1^2r_2^2}{q_{2+}r_{1-}}+\frac{q_2^2r_1^2}{q_{1+}r_{1-}}+\frac{q_2^2r_1^2}{q_{1+}r_{2-}}. \tag{28}$$

5. $\mathcal{A}_5$

We again present

$$X_5=(p\mathcal{A}^{(4)})=-C_5\frac{1}{t_1^2}Z_5 \tag{29}$$

and find

$$Z_5=(L_1L_2)=(a_1+b_1n^++c_1n^-,a_2+b_2n^++c_2n^-)$$
$$=(a_1a_2)+b_1a_{2+}+b_2a_{1+}+c_1a_{2-}+c_2a_{1-}+b_1c_2+b_2c_1 \tag{30}$$

Note that this is not the total contribution from $\mathcal{A}^{(5)}$ but only half of it containing $1/t_1^2$. The other half contains $-1/t_2^2$ and so has a different structure from the other amplitudes. It has to be taken into account in amplitudes with $1 \leftrightarrow 2$. Also note the sign "-" in (29). Calculations give

$$Z_5 = (a_1 a_2) - q_1^2 \frac{r_{2-}}{r_{1-}} - q_2^2 \frac{r_{1-}}{r_{2-}} - r_1^2 \frac{q_{2+}}{q_{1+}} - r_2^2 \frac{q_{1+}}{q_{2+}} + \frac{q_1^2 r_2^2}{q_{2+} r_{1-}} + \frac{q_2^2 r_1^2}{q_{1+} r_{2-}}. \tag{31}$$

6. $\mathcal{A}^{tot}$

Summing our contributions we find

$$X^{tot} = (p\mathcal{A}^{tot}) = g^3 \frac{1}{t_1^2} (\sum_{i=1}^{4} C_i Z_i - C_5 Z_5) = g^3 \frac{1}{t_1^2} \Big[C_1 (Z_1 + Z_4 - Z_5) + C_2 (Z_2 - Z_3 - Z_5) \Big] \tag{32}$$

3.2 $Z_1 + Z_4 - Z_5$

Since our expressions are long enough we separately consider terms with double poles at $q_{i+} = 0$ and $r_{i-} = 0$, $i = 1, 2$, simple poles and non-singular terms.

Suppressing the common factor g^3/t_1^2, the double pole contribution is

$$Z_{dp}^{(1+4-5)} = \Big(-\frac{q_2^2 r_1^2}{q_{1+} r_{1-}} \Big) + \Big(\frac{q_2^2 r_1^2}{q_{1+} r_{1-}} + \frac{q_2^2 r_1^2}{q_{1+} r_{2-}} + \frac{q_1^2 r_2^2}{q_{1+} r_{2-}} \Big) - \Big(\frac{q_1^2 r_2^2}{q_{2+} r_{1-}} + \frac{q_2^2 r_1^2}{q_{1+} r_{2-}} \Big) = 0,$$

where the 3 terms in the brackets correspond to contributions from Z_1, Z_4 and Z_5. Obviously they cancel.

The single pole contribution is

$$Z_p^{(1+4-5)} = \Big(r_1^2 \frac{q_{2+}}{q_{1+}} \Big) + \Big(-2r_1^2 \frac{q_{2+}}{q_{1+}} - r_2^2 \frac{q_{1+}}{q_{2+}} - q_1^2 \frac{r_{2-}}{r_{1-}} - q_2^2 \frac{r_{1-}}{r_{2-}} \Big) - \Big(-q_1^2 \frac{r_{2-}}{r_{1-}} - q_2^2 \frac{r_{1-}}{r_{2-}} - r_1^2 \frac{q_{2+}}{q_{1+}} - r_2^2 \frac{q_{1+}}{q_{2+}} \Big) = 0.$$

Again the three terms correspond to contributions from Z_1, Z_4, Z_5

The three terms with the nonsingular contributions are

$$Z_{ns}^{(1+4-5)} = \Big(-3r_{1-}(q_{1+} + q_{2+}) + (a_1, q_1 + 2q_2 - r_1) + q_1^2 + q_2^2 + 2r_1^2 - t_1^2 \Big)$$

$$+ \Big(3r_{1-}(q_{1+} + q_{2+}) - (a_1, q_1 + q_2 - r_1 - r_2) - q_1^2 - q_2^2 - 2r_1^2 \Big) - (a_1 a_2).$$

We find for vectors multiplying a_1 in the first two terms

$$q_1 + 2q_2 - r_1 - q_1 - q_2 + r_1 + r_2 = a_2$$

So in the sum all terms cancel except containing t_1^2: $Z_{ns}^{(1+4-5)} = -t_1^2$. So restoring the suppressed factor $X^{(1+4-5)} = -g^3 C_1$. This means that the sum of all diagrams considered above is not transversal by itself. Violation of transversality comes from the contribution $\mathcal{A}_1$.

3.3 $Z_2 - Z_3 - Z_5$

In a similar fashion here we find the double pole contribution

$$Z_{dp}^{(2+3-5)} = -\Big(\frac{q_1^2 r_2^2}{q_{1+} r_{1-}} \Big) + \Big(\frac{q_2^2 r_1^2}{q_{1+} r_{2-}} + \frac{q_1^2 r_2^2}{q_{1+} r_{1-}} + \frac{q_1^2 r_2^2}{q_{2+} r_{1-}} \Big) - \Big(\frac{q_1^2 r_2^2}{q_{2+} r_{1-}} + \frac{q_2^2 r_1^2}{q_{1+} r_{2-}} \Big) = 0.$$

The 3 terms in the brackets correspond to contributions from Z_2, Z_3 and Z_5. They cancel in the sum.

The single pole contribution is

$$Z_p^{(2-3-5)}=+\Big(q_1^2\frac{r_{2-}}{r_{1-}}\Big)-\Big(2q_1^2\frac{r_{2-}}{r_{1-}}+q_2^2\frac{r_{1-}}{r_{2-}}+r_1^2\frac{q_{2+}}{q_{1+}}+r_2^2\frac{q_{1+}}{q_{2+}}\Big)$$

$$-\Big(-q_1^2\frac{r_{2-}}{r_{1-}}-q_2^2\frac{r_{1-}}{r_{2-}}-r_1^2\frac{q_{2+}}{q_{1+}}-r_2^2\frac{q_{1+}}{q_{2+}}\Big).$$

They also give zero in the sum.

Finally the nonsingular contribution is

$$Z_{ns}^{(2-3-5)}=+\Big(-3q_{1+}(r_{1-}+r_{2-})-(a_1,q_1-r_1-2r_2)+r_1^2+r_2^2+2q_1^2-t_1^2\Big)$$

$$-\Big(-3q_{1+}(r_{1-}+r_{2-})-(a_1,q_1+q_2-r_1-r_2)+r_1^2+r_2^2+2q_1^2\Big)-(a_1a_2).$$

We find for vectors multiplying a_1 in the first 2 terms

$$-(q_1-r_1-2r_2-q_1-q_2+r_1+r_2)=a_2.$$

In the sum all terms cancel again except containing t_1^2: $Z_{ns}^{(2-3-5)}=-t_1^2$. So restoring the factor in front $X^{(2-3-5)}=-g^3C_2$. Again the sum of all diagrams considered above is not transversal by itself. In this part violation of transversality comes from contributions $\mathcal{A}_2$.

For the sum of all diagrams with fixed reggeon momenta we find

$$X^{tot}=-g^3(C_1+C_2)=-g^3C_5. \tag{33}$$

This expression is antisymmetric under interchange $(a_2,a_1|b_2,b_1)\leftrightarrow(a_1,a_2|b_1,b_2)$ and does not depend on the momenta of the 4 reggeons. So it will vanish after symmetrization in the sum $\mathcal{A}(2,1|2,1)+\mathcal{A}(1,2|1,2)$. Thus after symmetrization we find transversality of the constructed RR$\to$RRP vertex.

4 On-mass-shell amplitudes

On mass shell, at $p^2=0$, the physical amplitudes can be presented via the physical polarization vector ϵ_μ, which we choose with the properties

$$(p\epsilon)=(l\epsilon)=0,\quad \epsilon_+=0,\quad \epsilon_-=-\frac{(p\epsilon)_\perp}{p_+}. \tag{34}$$

So the product with any vector v

$$(v\epsilon)=(v\epsilon)_\perp-\frac{v_+}{p_+}(p\epsilon)_\perp. \tag{35}$$

Amplitudes $\mathcal{A}_i$, $i=1,...,5$ take the following form on the mass shell multiplied by the polarization vector ϵ

1. $\mathcal{A}_1$

Obviously it is sufficient to transform our coefficients $a,b,...,e$. We have

$$a_\epsilon\equiv(a\epsilon)=0,$$

$$b_\epsilon\equiv(b\epsilon)=2p_+(q_1\epsilon)+q_{1+}(p-2t,\epsilon).$$

Since $p-2t=-p-2r_2$, $(p-2t,\epsilon)=-2(r_2\epsilon)$, which gives

$$b_\epsilon=2p_+\Big((q_1\epsilon)_\perp-\frac{q_{1+}}{p_+}(p\epsilon)_\perp\Big)-2q_{1+}(r_2\epsilon)_\perp=2p_+(q_1\epsilon)_\perp-2q_{1+}(p+r_2,\epsilon)_\perp.$$

Similarly,

$$c_\epsilon = 2p_+\Big((q_2\epsilon)_\perp - \frac{q_{2+}}{p_+}(p\epsilon)_\perp\Big) - 2q_{2+}(r_2\epsilon)_\perp = 2p_+(q_2\epsilon)_\perp - 2q_{2+}(p+r_2,\epsilon)_\perp.$$

and finally

$$e_\epsilon = -2(r_2\epsilon)_\perp - 2(p\epsilon)_\perp.$$

2. $\mathcal{A}_2$

Again we transform coefficients $\bar{a}, ..., \bar{e}$. We have

$$\bar{a}_\epsilon = (p\epsilon)_\perp \frac{t^2}{p_+},$$

$$\bar{b}_\epsilon = 2p_-(r_1\epsilon) + r_{1-}(p-2t,\epsilon) + \frac{(p\epsilon)_\perp}{p_+}\Big(2(pr_1) + q_2^2\frac{r_{1-}}{p_-}\Big),$$

where $t = p - q_2$, so that $(p-2t,\epsilon) = 2(q_2\epsilon)$. We find

$$\bar{b}_\epsilon = 2p_-(r_1\epsilon)_\perp + 2r_{1-}\Big((q_2\epsilon)_\perp - (p\epsilon)_\perp\frac{q_{2+}}{p_+}\Big) + 2(p\epsilon)_\perp\Big(r_{1-} - r_{1-}\frac{q_2^2}{p_\perp^2} + \frac{(pr_1)_\perp}{p_+}\Big).$$

Similarly

$$\bar{c}_\epsilon = 2p_-(r_2\epsilon)_\perp + 2r_{2-}\Big((q_2\epsilon)_\perp - (p\epsilon)_\perp\frac{q_{2+}}{p_+}\Big) + 2(p\epsilon)_\perp\Big(r_{2-} - r_{2-}\frac{q_2^2}{p_\perp^2} + \frac{(pr_2)_\perp}{p_+}\Big).$$

Finally

$$\bar{e}_\epsilon = 2(q_2\epsilon)_\perp + 2(p\epsilon)_\perp\Big(1 - \frac{q_{2+}}{p_+} - \frac{q_2^2}{p_\perp^2}\Big).$$

3. $\mathcal{A}_3$

We present

$$\mathcal{A}_{3\epsilon} = g^3 C_3 \frac{1}{t_1^2} B_3, \tag{36}$$

where

$$B_3 = 2(p\epsilon)_\perp\frac{a_{1+}}{p_+} - (a_1\epsilon)_\perp + 2\frac{(p\epsilon)_\perp}{p_+}\Big(\frac{q_1^2}{r_{1-}} - 2q_{1+}\Big) - 2\frac{(p\epsilon)_\perp q_2^2}{p_\perp^2}\Big(\frac{r_1^2}{q_{1+}} - r_{1-}\Big).$$

Somewhat transforming we find

$$B_3 = -(a_1\epsilon)_\perp + 2(p\epsilon)_\perp\Big[-\frac{q_{1+}}{p_+} + \frac{q_1^2}{p_+r_{1-}} - \frac{q_2^2}{p_\perp^2 r_{2-}}\Big(\frac{r_1^2}{q_{1+}} - r_{1-}\Big)\Big],$$

where $a_1 = q_1 + r_1$

4. $\mathcal{A}_4$

We present

$$\mathcal{A}_{4\epsilon} = g^3 C_4 \frac{1}{t_1^2} B_4, \tag{37}$$

where

$$B_4 = -(p\epsilon)_\perp\frac{q_{1+}}{p_+} - (a_1\epsilon)_\perp - \frac{(p\epsilon)_\perp}{p_+}\Big(\frac{q_1^2}{r_{1-}} - 2q_{1+}\Big) + \frac{(p\epsilon)_\perp q_2^2}{p_+r_{1-}r_{2-}}\Big(\frac{r_1^2}{q_{1+}} - r_{1-}\Big).$$

Again transforming to obtain final expresion

$$B_4 = -(a_1\epsilon)_\perp + \frac{(p\epsilon)_\perp}{p_+}\Big(q_{1+} - \frac{q_1^2}{r_{1-}} - \frac{q_2^2}{r_{2-}} + \frac{q_2^2 r_1^2}{q_{1+}r_{1-}r_{2-}}\Big).$$

5. $\mathcal{A}_5$
Multiplying by ϵ we find

$$L_{1\epsilon} = (L_1\epsilon) = (a_1\epsilon)_\perp - \frac{(p\epsilon)_\perp}{p_+}\Big(\frac{q_1^2}{r_{1-}} - q_{1+}\Big),$$

$$L_{2\epsilon} = (L_2\epsilon) = (a_2\epsilon)_\perp - \frac{(p\epsilon)_\perp}{p_+}\Big(\frac{q_2^2}{r_{2-}} - q_{2+}\Big)$$

and finally

$$(t_1 - t_2)_\epsilon = (t_1 - t_2, \epsilon) = (t_1 - t_2, \epsilon)_\perp - \frac{(p\epsilon)_\perp}{p_+}(q_{1+} - q_{2+}).$$

The product $(\mathcal{A}_5\epsilon)$ will be given by Eq. (18) with vectors substituted by their products with ϵ given above.

5 Asymptotics for large q_{1+} or r_{1-} with fixed p

For applications the behaviour of the vertex at large values of longitudinal momenta has the utter importance, since one has to integrate over them when the vertex is inserted into the amplitude. The necessary condition for the possibility of this integration is that the amplitude should vanish at high values of longitudinal momenta. Note that in the inclusive cross-section momentum p of the observed gluon is fixed. This means that sums $q_{1+} + q_{2+}$ and $r_{1-} + r_{2-}$ remain finite when one of the longitudinal momenta tends to infinity. Having in mind that in the D-ND configuration upper and lower reggeons enter in the different manner we have to study separately cases of $q_{1+} \to \infty$ and $r_{1-} \to \infty$.

5.1 $q_{1+} \to \infty$, p fixed

We consider subsequently our amplitudes $\mathcal{A}_i$, $i = 1, ...5$.
1. $\mathcal{A}_1$
Here $t = q_1 + q_2 - r_1$. Since $q_2 = p - q_1$, t is finite. Of the two denominators one is finite the other grows as q_{1+}. So non-vanishing terms come from the ones in the numerator which grow as q_{1+} or faster. Inspecting coefficients a, b, c, e we conclude: a and e are finite,

$$b = 2p_+q_1 + q_{1+}(p - 2t) - q_{1+}n^+\Big(2p_- + \frac{r_2^2}{p_+}\Big),$$

$$c = 2p_+q_2 + q_{2+}(p - 2t) - q_{2+}n^+\Big(2p_- + \frac{r_2^2}{p_+}\Big).$$

Turning to $\bar{A}, ...\bar{E}$ we find that $\bar{A}, \bar{B}$ are finite with $\bar{B} = \bar{C}$. So the contribution $a\bar{A} - b\bar{B} - c\bar{C}$ is finite. We are left with only $e\bar{E}$.

At large q_{1+} $\bar{E} = \bar{E}_0 - 2q_{2+}r_{1-}$, where

$$\bar{E}_0 = (r_1 + q_1, t + q_2) + q_2^2 - q_1^2 + (q_1 - r_1)^2 + 2r_{1-}q_{1+} = r_{1-}q_{2+} - r_{1-}q_{1+}$$

We find $\bar{E} = -r_{1-}(q_{1+} + q_{2+})$ and is finite. So there are no growing terms in the numerator and in the limit $q_{1+} \to \infty$ $\mathcal{A}_1 = 0$.

2. $\mathcal{A}_2$

Here $\bar{t} = q_1 - r_1 - r_2$ and grows as q_{1+}. The two denominators both grow as q_{1+}. Possible terms non-vanishing at $q_{1+} \to \infty$ may come from the ones in the numerator growing as q_{1+}^2 or faster.

Coefficients $\bar{a}, ...\bar{e}$ are in this limit

$$\bar{a} = n^- \bar{t}^2, \quad \bar{b} = -2\bar{t}r_{1-}, \quad \bar{c} = -3\bar{t}r_{2-}, \quad \bar{e} = -2\bar{t}.$$

Terms $A, ...E$ are

$$A = 3q_{1+}, \quad B = C = 4q_{1+}, \quad E = E_0 + 2q_{1+}(\bar{t}_- - r_{2-}),$$

where

$$E_0 = -(q_1 + r_1, \bar{t} - r_2) + r_2^2 - r_1^2 + (q_1 - r_1)^2 + 2q_{1+}r_{1-} = -q_{1+}(\bar{t} - r_2)_- - q_{1+}r_{1-}.$$

So

$$E = q_{1+}(\bar{t} - r_{2-} - r_{1-}) = -2q_{1+}(r_{1-} + r_{2-}).$$

Thus

$$\bar{a}A + \bar{b}B + \bar{c}C + \bar{e}E = 6q_{1+}^2(r_{1-} + r_{2-}) - 2q_{1+}^2 r_{1-} - 2q_{1+}^2 r_{2-} + 4q_{1+}^2(r_{1-} + r_{2-}) = 2q_{1+}(r_{1-} + r_{2-}).$$

The denominator is $\bar{t}^2 t_1^2 = 4q_{1+}^2 r_{1-}(r_{1-} + r_{2-})$. Thus in the limit $q_{1+} \to \infty$

$$\mathcal{A}_{2+} = -g^3 C_2 \frac{1}{2r_{1-}}. \tag{38}$$

3. $\mathcal{A}_3$

In the limit $q_{1+} \to \infty$ the square bracket in (13) is $-a_+ n^- - a + 4n^- q_{1+}$. So the growing "+" component is just $2q_{1+}$ and in the limit

$$\mathcal{A}_{3+} = -g^3 C_3 \frac{1}{r_{1-}}. \tag{39}$$

4. $\mathcal{A}_4$

In the limit $q_{1+} \to \infty$ the square bracket in (14)is $2q_{1+}n^- - a + n^- q_{1+}$. So the growing "+" component is just $-q_{1+}$ and in this limit

$$\mathcal{A}_{4+} = g^3 C_4 \frac{1}{2r_{1-}}. \tag{40}$$

5. $\mathcal{A}_5$

The two denominators grow as q_{1+} each. So we have to search for terms in the numerator which grow as q_{1+}^2 or faster. We have

$$(t_2 L_1) = (q_2 - r_2, q_1 + r_1) + b_1 q_{2+} - c_1 r_{2-}$$

$$= q_{2+}r_{1-} - q_{1+}r_{2-} - 2q_{2+}r_{1-} + 2q_{1+}r_{2-} = q_{1+}r_{2-} - q_{2+}r_{1-}.$$

We also have in the limit $q_{1+} \to \infty$ $L^{(2)} = a_2 - 2n^- q_{1+}$, so that the growing "+" component is $L_{2+} = -q_{1+}$. As a result

$$L_{2+}(t_2 L_1) - (1 \leftrightarrow 2) = q_{1+}(q_{2+}r_{1-} - q_{1+}r_{2-}) - q_{2+}(q_{1+}r_{2-} - q_{2+}r_{1-})$$

$$= q_{1+}(q_{2+}r_{1-} - q_{1+}r_{2-} + q_{1+}r_{2-} - q_{2+}r_{1-}) = 0.$$

So the terms growing with q_{1+} come from the third term in (18). We find in the limit $q_{1+}\to\infty$

$$(L_1L_2)=(a_1a_2)+b_1a_{2+}+b_2a_{1+}+c_1a_{2-}+c_2a_{1-}+b_1c_2+b_2c_1$$
$$=q_{1+}r_{2-}+q_{2+}r_{1-}-2q_{1+}r_{2-}-2q_{2+}r_{1-}-2q_{1+}r_{2-}-2q_{2+}r_{1-}+4q_{1+}r_{2-}+4q_{2+}r_{1-}$$
$$=q_{1+}r_{2-}+q_{2+}r_{1-}.$$

So we find the "+"component

$$(t_{1+}-t_{2+})(L^{(1)}L^{(2)})=2q_{1+}(q_{1+}r_{2-}+q_{2+}r_{1-})=2q_{1+}^2(r_{2-}-r_{1-})$$

and thus

$$\mathcal{A}_{5+}=g^3C_5\Big(\frac{1}{2r_{1-}}-\frac{1}{2r_{2-}}\Big). \tag{41}$$

6.

After addition of the symmetrized contributions we find for "+" components suppressing the common factor $g^3f^{a_2a_1c}$ or $g^3f^{a_2b_1c}$ for D-ND and ND-ND configurations respectively:

$$\mathcal{A}_2^{tot}=-(\kappa_2^{(1)}-\kappa_2^{(3)})\frac{1}{2r_{1-}}-(\kappa_2^{(2)}-\kappa_2^{(4)})\frac{1}{2r_{2-}},\quad \mathcal{A}_3^{tot}=-(\kappa_3^{(1)}-\kappa_3^{(3)})\frac{1}{r_{1-}}-(\kappa_3^{(2)}-\kappa_3^{(4)})\frac{1}{r_{2-}},$$

$$\mathcal{A}_4^{tot}=(\kappa_4^{(1)}-\kappa_4^{(3)})\frac{1}{2r_{1-}}+(\kappa_2^{(2)}-\kappa_2^{(4)})\frac{1}{2r_{2-}},\quad \mathcal{A}_5^{tot}=\frac{1}{2}\Big(\kappa_5^{(1)}-\kappa_5^{(3)}\Big)\Big(\frac{1}{r_{1-}}-\frac{1}{r_{2-}}\Big).$$

For D-ND configuration we have shown

$$\kappa_2^{(1)}=\kappa_2^{(3)}=-\kappa_2^{(2)}=-\kappa_2^{(4)}=1,\quad \kappa_3^{(1)}=\kappa_3^{(3)}=-\kappa_3^{(2)}=-\kappa_3^{(4)}=-1,$$

$$\kappa_4^{(1)}=\kappa_4^{(3)}=-\kappa_4^{(2)}=-\kappa_4^{(4)}=-\frac{1}{2},\quad \kappa_5^{(1)}=\kappa_5^{(3)}=-\kappa_5^{(2)}=-\kappa_5^{(4)}=\frac{1}{2}.$$

So all contributions are zero. This means that each of the diagrams studied above separately behaves as $1/q_{1+}$ as $q_{1+}\to\infty$.

For ND-ND configuration, using $\kappa_i^{(k)}$ from the table we find

$$\mathcal{A}_2^{tot}=\Big(\frac{1}{2}+\frac{1}{2}\Big)\frac{1}{r_{1-}}+\frac{1}{4}\frac{1}{r_{2-}},\quad \mathcal{A}_3^{tot}=\Big(-1-1\Big)\frac{1}{r_{1-}}-\frac{1}{2}\frac{1}{r_{2-}},$$

$$\mathcal{A}_4^{tot}=\Big(\frac{1}{4}+\frac{1}{2}\Big)\frac{1}{r_{1-}}+\frac{1}{2}\frac{1}{r_{2-}},\quad \mathcal{A}_5^{tot}=\frac{1}{4}\frac{1}{r_{1-}}-\frac{1}{4}\frac{1}{r_{2-}}.$$

Subsequent terms on the right-hand side correspond to contributions from $i=1,3$ and 4. We observe that in this case separate contributions do not vanish in the limit $q_{1+}\to\infty$. However, the sum of them does vanish in this limit.

5.2 $r_{1-}\to\infty$, p fixed

Again we subsequently study contributions $\mathcal{A}_i$, $i=1,...5$.

1. $\mathcal{A}_1$

In this case $t=q_1+q_2-r_1$ grows and the two denominators each grow as r_{1-}. So we have to separate terms in the numerator growing as r_{1-}^2.

Coefficients $a,...e$ are at large t

$$a=-n^+t^2,\quad b=-2tq_{1+},\quad c=-2tq_{2+},\quad e=-2t.$$

Terms $\bar{A}, ...\bar{E}$ are

$$\bar{A} = 3t_-, \quad \bar{B} = \bar{C} = 4t_-, \quad \bar{E} = \bar{E}_0 - 2r_{1-}(t_+ + q_{2+}),$$

where

$$\bar{E}_0 = (r_1 + q_1, t + q_2) - (r_1 - q_1)^2 - 2r_{1-}q_{1+} = r_{1-}(t_+ + q_{2+}) - r_{1-}q_{1+}.$$

As a result

$$\bar{E} = -r_{1-}(t_+ + q_{2+}) - r_{1-}q_{1+} = -2r_{1-}(q_{1+} + q_{2+}).$$

Thus the growing "-" component is

$$a_-\bar{A} - b_-\bar{B} - c_-\bar{C} + e_-\bar{E} = -3t^2t_- + 8t_-^2 q_{1+} + 8t_-^2 q_{2+} - 4t_-^2(q_{1+} + q_{2+}) == -2t_-^2(q_{1+} + q_{2+}).$$

The denominator is $4t_-^2 q_{1+}(q_{1+} + q_{2+})$ so that finally

$$\mathcal{A}_{1-} = g^3 C_1 \frac{1}{2q_{1+}}. \tag{42}$$

2. $\mathcal{A}_2$

Here $\bar{t} = q_1 - r_1 - r_2$ and is finite. As a result coefficients $\bar{a}$ and $\bar{e}$ are finite. The rest

$$\bar{b} = 2p_- r_1 + r_{1-}(p - 2\bar{t}) - n^- r_{1-}\Big(2p_+ + \frac{q_2^2}{p_-}\Big),$$

$$\bar{c} = 2p_- r_2 + r_{2-}(p - 2\bar{t}) - n^- r_{2-}\Big(2p_+ + \frac{q_2^2}{p_-}\Big).$$

Terms $A, ...E$ are

$$A = 3q_{1+}, \quad B = C = 4q_{1+}, \quad E = E_0 + 2q_{1+}(\bar{t}_- - r_{2-}),$$

where

$$E_0 = -(q_1 + r_1)(\bar{t} - r_2) - (q_1 - r_1)^2 - 2q_{1+}r_{1-} = -q_{1+}(\bar{t}_- - r_{2-}) - q_{1+}r_{1-}.$$

So

$$E = q_{1+}(\bar{t}_- - r_{2-}) - q_{1+}r_{1-} = 2q_{1+}p_-$$

and is finite. We observe that all growing terms cancel and in the limit $r_{1-} \to \infty$ $\mathcal{A}_2 = 0$.

3. $\mathcal{A}_3$

In the limit $r_{1-} \to \infty$ the square bracket in (13) is $2r_{1-}n^+ - r_1 - 2n^+ r_{1-}$. So the growing "-" component is $-r_{1-}$ and in this limit

$$\mathcal{A}_{3-} = g^3 C_3 \frac{1}{2q_{1+}}. \tag{43}$$

4. $\mathcal{A}_4$

In the limit $r_{1-} \to \infty$ the square bracket in (13) is $-r_{1-}n^+ - a + 4n^+ r_{1-}$ So the growing "-" component is $2r_{1-}$ and in this limit

$$\mathcal{A}_{4-} = -g^3 C_4 \frac{1}{q_{1+}}. \tag{44}$$

5. $\mathcal{A}_5$

The two denominators grow as r_{1-} each. So we have to search for terms in the numerators which grow as r_{1-}^2 or faster. We have seen that

$$(t_2 L_1) = (q_2 - r_2, q_1 + r_1) + b_1 q_{2+} - c_1 r_{2-}$$

$$= q_{2+}r_{1-} - q_{1+}r_{2-} - 2q_{2+}r_{1-} + 2q_{1+}r_{2-} = q_{1+}r_{2-} - q_{2+}r_{1-}.$$

We also have in the limit $r_{1-} \to \infty$ $L_2 = r_2 - 2n^+ r_{1-}$, so that the growing "-" component is $L_+^{(2)} = -r_{1-}$. As a result

$$L_{2-}(t_2L_1) - (1 \leftrightarrow 2) = r_{1-}(q_{2+}r_{1-} - q_{1+}r_{2-}) - r_{2-}(q_{1+}r_{2-} - q_{2+}r_{1-})$$

$$= r_{1-}(q_{2+}r_{1-} - q_{1+}r_{2-} + q_{1+}r_{2-} - q_{2+}r_{1-}) = 0.$$

So the terms growing with r_{1-} come again from the third term in (18). We find in the limit $r_{1-} \to \infty$

$$(L_1L_2) = (a_1a_2) + b_1a_{2+} + b_2a_{1+} + c_1a_{2-} + c_2a_{1-} + b_1c_2 + b_2c_1$$

$$= q_{1+}r_{2-} + q_{2+}r_{1-} - 2q_{1+}r_{2-} - 2q_{2+}r_{1-} - 2q_{1+}r_{2-} - 2q_{2+}r_{1-} + 4q_{1+}r_{2-} + 4q_{2+}r_{1-}$$

$$= q_{1+}r_{2-} + q_{2+}r_{1-}$$

So we find the "-"component

$$(t_{1-} - t_{2-})(L^{(1)}L^{(2)}) = -2r_{1-}(q_{1+}r_{2-} + q_{2+}r_{1-}) = 2r_{1-}^2(q_{1+} - q_{2+})$$

and as a result

$$\mathcal{A}_-^{(5)} = g^3 C_5\Big(\frac{1}{2q_{1+}} - \frac{1}{8q_{2+}}\Big). \tag{45}$$

6.

After addition of the symmetrized contributions and suppressing the common factors as before we find for "-" components

$$\mathcal{A}_1^{tot} = (\kappa_1^{(1)} + \kappa_1^{(2)})\frac{1}{2q_{1+}} + (\kappa_1^{(3)} + \kappa_1^{(4)})\frac{1}{2q_{2+}}, \quad \mathcal{A}_3^{tot} = (\kappa_3^{(1)} + \kappa_3^{(2)})\frac{1}{2q_{1+}} + (\kappa_3^{(3)} + \kappa_3^{(4)})\frac{1}{2q_{2+}},$$

$$\mathcal{A}_4^{tot} = -\Big((\kappa_4^{(1)} + \kappa_4^{(2)})\frac{1}{q_{1+}} - (\kappa_4^{(3)} + \kappa_4^{(4)})\frac{1}{q_{2+}}\Big), \quad \mathcal{A}_5^{tot} = \frac{1}{2}(\kappa_5^{(1)} + \kappa_5^{(2)})\Big(\frac{1}{q_{1+}} - \frac{1}{q_{2+}}\Big).$$

For D-ND configuration

$$\kappa_1^{(1)} + \kappa_1^{(2)} = -1, \quad \kappa_3^{(1)} + \kappa_3^{(2)} = -2, \quad \kappa_4^{(1)} + \kappa_4^{(2)} = -1, \quad \kappa_5^{(1)} + \kappa_5^{(2)} = 1$$

and for all i

$$\kappa_i^{(3)} + \kappa_i^{(4)} = -\kappa_i^{(1)} - \kappa_i^{(2)}$$

So we get

$$\mathcal{A}_1^{tot} = -\frac{1}{2}\Big(\frac{1}{q_{1+}} - \frac{1}{q_{2+}}\Big), \quad \mathcal{A}_3^{tot} = -\Big(\frac{1}{q_{1+}} - \frac{1}{q_{2+}}\Big),$$

$$\mathcal{A}_4^{tot} = \Big(\frac{1}{q_{1+}} - \frac{1}{q_{2+}}\Big), \quad \mathcal{A}_5^{tot} = \frac{1}{2}\Big(\frac{1}{q_{1+}} - \frac{1}{q_{2+}}\Big).$$

So unlike the limit $q_{1+} \to \infty$ in this configuration the individual contributions do not vanish in the limit $r_{1-} \to \infty$. However, their sum vanishes.

For ND-ND configuration we find using the Table

$$\mathcal{A}_1^{tot} = \frac{1}{2}\Big(\frac{1}{2}\frac{1}{q_{1+}} + 2\frac{1}{q_{2+}}\Big), \quad \mathcal{A}_3^{tot} = \frac{1}{2}\Big(\frac{1}{q_{1+}} + \frac{3}{2}\frac{1}{q_{2+}}\Big),$$

$$\mathcal{A}_4^{tot} = -\Big(\frac{1}{2}\frac{1}{q_{1+}} + 2\frac{1}{q_{2+}}\Big), \quad \mathcal{A}_5^{tot} = -\frac{1}{4}\Big(\frac{1}{q_{1+}} - \frac{1}{q_{2+}}\Big)$$

The individual contributions do not vanish again. Their sum is

$$\mathcal{A}^{tot} = \sum_{i=1}^{5} \mathcal{A}_i^{tot} = \frac{1}{q_{1+}}\Big(\frac{1}{4} + \frac{1}{2} - \frac{1}{2} - \frac{1}{4}\Big) + \frac{1}{q_{2+}}\Big(1 + \frac{3}{4} - 2 + \frac{1}{4}\Big) = 0$$

So it vanishes at $r_{1-} \to \infty$.

Note that at $r_{1-} \to \infty$ the leading contribution comes from the "-" component of the vertex. So one may expect still better convergence for the amplitude on the mass shell multiplied by the polarization vector ϵ with a zero "+" component. Numerical calculations show that this is indeed so in the D-ND configuration due to cancellations in the sum with interchanged r_{1-} and r_{2-}. In this case the vertex behaves as $1/r_{1-}^2$ at $r_{1-} \to \infty$. However, the same numerical calculations show that this result does not hold for the ND-ND configuration, in which the amplitude behaves as $1/r_{1-}$.

6 Poles in longitudinal momenta

Here we present contributions with poles at $q_{1+}, q_{2+}, r_{1-}, r_{2-} = 0$ coming from the induced vertices in the effective action formalism. In the case of gluon production in the collision of a single projectile on several targets these poles cancel with the singularities coming from rescattering contributions [16, 17, 18]. In our case there is no rescattering and one might think that these poles cancel in the total amplitude after taking into account all permutations of interacting reggeons. This possibility was advocated in [13] for the second order odderon kernel. However, we shall see that in our case pole singularities do not cancel and remain in the total production amplitude. For applications this means that one has to fix somehow the way to do longitudinal integrations in presence of these poles. The requirement of the hermiticity of effective action and the structure of the simple reggeon exchange prompt using integrations in the principal value sense.

Due to the complicated form of the production amplitudes the simplest way to see existence of pole singularities at $q_{1+}, q_{2+}, r_{1-}r_{2-} = 0$ is by a numerical check. It indeed shows that in both N-ND and ND-ND configurations the production amplitude contains pole singularities at each q_{1+}, q_{2+}, r_{1-} and r_{2-} equal to zero and also double pole singularities at, say, $q_{1+} = r_{1-} = 0$.

However, it is instructive to extract pole contributions in the analytical form to see their character and understand why they cannot cancel. We shall subsequently consider pole contributions from amplitudes $\mathcal{A}_i$ $i = 1, ...5$.

1. $\mathcal{A}_1$

Coefficients $a, ...e$ are not singular. Singular terms in $\bar{A}, ...\bar{E}$ are

$$\bar{A} = \frac{r_1^2}{q_{1+}}, \quad \bar{B} = 0, \quad \bar{C} = 2\frac{r_1^2}{q_{1+}}, \quad \bar{E} = 2\frac{r_1^2 q_{2+}}{q_{1+}} - \frac{q_2^2 r_1^2}{q_{1+}r_{1-}}.$$

Here $t = q_1 + q_2 - r_1$ Presenting the pole part of $\mathcal{A}_1$ as

$$\mathcal{A}_1 = -g^3 C_1 \frac{1}{t^2 t_1^2} X_1 \tag{46}$$

we find after trivial calculations

$$X_{1\mu} = \frac{r_1^2}{q_{1+}}\Big[p_+(p_\mu - 4q_{2\mu}) + n_\mu^+(4(pq_2) - 4p_- q_{2+} - t^2 - p^2) + 4p_+ q_{2+} n_\mu^-\Big]$$
$$- \frac{q_2^2 r_1^2}{q_{1+}r_{1-}}\Big[(p - 2t)_\mu - n_\mu^+\Big(2p_- + \frac{r_2^2}{p_+}\Big) + 2p_+ n_\mu^-\Big] \tag{47}$$

On mass shell, multiplied by the polarization vector, it gives

$$X_{1\epsilon}=-4(q_2\epsilon)_\perp\frac{r_1^2q_{2+}}{q_{1+}}+2(p+r_2,\epsilon)_\perp\frac{q_2^2r_1^2}{q_{1+}r_{1-}}. \tag{48}$$

2. $\mathcal{A}_2$

Coefficients $\bar a,...\bar e$ are not singular. Singular terms in $A,...E$ are

$$A=-\frac{q_1^2}{r_{1-}},\quad B=0,\quad C=-2\frac{q_1^2}{r_{1-}},\quad E=2r_{2-}\frac{q_1^2}{r_{1-}}-\frac{q_1^2r_2^2}{q_{1+}r_{1-}}.$$

Here $\bar t=q_1-r_1-r_2$. We present

$$\mathcal{A}_2=-g^3C_2\frac{1}{\bar t^2t_1^2}X_2. \tag{49}$$

Calculations give

$$X_{2\mu}=\frac{q_1^2}{r_{1-}}\Big[-p_-(p_\mu+r_{2\mu})+4p_-r_{2-}n_\mu^++n_\mu^-(\bar t^2+p^2+4(pr_2)-4p_+r_{2-}\Big]$$
$$-\frac{q_1^2r_2^2}{q_{1+}r_{1-}}\Big[(p-2\bar t)_\mu+2p_-n_\mu^+-n_\mu^-\Big(2p_++\frac{q_2^2}{p_-}\Big)\Big]. \tag{50}$$

On mass shell, multiplied by the polarization vector,

$$X_{2\epsilon}=-\frac{q_1^2}{r_{1-}}\Big[(p\epsilon)_\perp\Big(2r_{2-}+\frac{2(p,2r_2-q_2)_\perp+q_2^2}{p_+}\Big)+4p_-(r_2\epsilon)_\perp\Big]$$
$$-2\frac{q_1^2r_2^2}{q_{1+}r_{1-}}\Big[(q_2\epsilon)_\perp+(p\epsilon)_\perp\Big(\frac{q_{1+}}{p_+}-\frac{q_2^2}{p_\perp^2}\Big)\Big]. \tag{51}$$

3. $\mathcal{A}_3$

As before we present

$$\mathcal{A}_3=g^3C_3\frac{1}{t_1^2}X_3. \tag{52}$$

We find

$$X_{3\mu}=n_\mu^+\Big(\frac{r_1^2}{q_{1+}}+\frac{r_1^2}{q_{2+}}-\frac{q_1^2r_1^2}{q_{1+}q_{2+}r_{1-}}\Big)+n_\mu^-\Big(\frac{q_2^2r_{1-}}{r_{2-}p_-}-2\frac{q_1^2}{r_{1-}}-\frac{q_2^2r_1^2}{q_{1+}r_{2-}p_-}\Big). \tag{53}$$

On mass shell, multiplied by the polarization vector,

$$X_{3\epsilon}=2(p\epsilon)_\perp\Big(\frac{q_1^2}{p_+r_{1-}}+\frac{q_2^2r_{1-}}{r_{2-}p_\perp^2}-\frac{q_2^2r_1^2}{q_{1+}r_{2-}p_\perp^2}\Big). \tag{54}$$

4. $\mathcal{A}_4$

Again we present

$$\mathcal{A}_4=g^3C_4\frac{1}{t_1^2}X_4. \tag{55}$$

We find

$$X_{4\mu}=-n_\mu^+\Big(2\frac{r_1^2}{q_{1+}}+\frac{r_1^2q_{1+}}{p_+q_{2+}}-\frac{q_1^2r_1^2}{p_+q_{2+}r_{1-}}\Big)+n_\mu^-\Big(\frac{q_1^2}{r_{1-}}+\frac{q_2^2}{r_{2-}}-\frac{q_2^2r_1^2}{q_{1+}r_{1-}r_{2-}}\Big). \tag{56}$$

On mass shell, multiplied by the polarization vector,

$$X_{4\epsilon}=-\frac{(p\epsilon)_\perp}{p_+}\Big(\frac{q_1^2}{r_{1-}}+\frac{q_2^2}{r_{2-}}-\frac{q_2^2r_1^2}{q_{1+}r_{1-}r_{2-}}\Big). \tag{57}$$

5. $\mathcal{A}_5$

Presenting

$$\mathcal{A}_5 = g^3 C_5 \frac{1}{t_1^2 t_2^2} X_5, \tag{58}$$

we use Eq. (59). To simplify quite cumbersome expressions we from the start restrict ourselves to mass-shell and multiply $\mathcal{A}_5$ by the polarization vector. Singular terms are contained in $A^{(i)}$, $i = 1, 2, 3, 5$ (term with $A^{(4)}$ drops out in our gauge). We find

$$A^{(1)} = 2\frac{q_2^2 r_{1-}}{r_{2-}} - 2\frac{r_2^2 q_{1+}}{q_{2+}}, \quad A^{(2)} = -2\frac{q_1^2 r_{2-}}{r_{1-}} + 2\frac{r_1^2 q_{2+}}{q_{1+}},$$

$$A^{(3)} = -\frac{q_1^2 r_{2-}}{r_{1-}} - \frac{r_1^2 q_{2+}}{q_{1+}} - \frac{q_2^2 r_{1-}}{r_{2-}} - \frac{r_2^2 q_{1+}}{q_{2+}} + \frac{q_1^2 r_2^2}{q_{2+} r_{1-}} + \frac{q_2^2 r_1^2}{q_{1+} r_{2-}}$$

and the most complicated term

$$A^{(5)} = \frac{q_2^2}{r_{2-}}(a^{(1)}, p + t_2) + \frac{q_1^2 q_2^2}{r_{2-}} + \frac{q_2^2 r_1^2}{r_{2-}}$$

$$+4\frac{q_1^2 q_{1+} r_{2-}}{r_{1-}} + 4\frac{r_2^2 q_{1+}^2}{q_{2+}} + 4\frac{q_1^2 q_{2+} r_{2-}}{r_{1-}} + 2\frac{q_2^2 r_1^2 q_{2+}}{q_{1+} r_{2-}} - (1 \leftrightarrow 2),$$

where $a^{(1)} = q_1 + r_1$ and $a^{(2)} = q_2 + r_2$ The coefficients are

$$(a^{(1)}\epsilon) = (a^{(1)}, \epsilon)_\perp - (p\epsilon)_\perp \frac{q_{1+}}{p_+}, \quad (a^{(2)}\epsilon) = (a^{(2)}, \epsilon)_\perp - (p\epsilon)_\perp \frac{q_{2+}}{p_+},$$

$$(\tau^{(3)}\epsilon) = (q_1 - r_1 - q_2 + r_2, \epsilon)_\perp - (p\epsilon)_\perp \frac{q_{1+} - q_{2+}}{p_+}.$$

Collecting all terms we find

$$X_{5\epsilon} = (a^{(1)}\epsilon)_\perp A^{(1)} + (a^{(2)}\epsilon)_\perp A^{(2)} + (t_1 - t_2, \epsilon)_\perp A^{(3)} - \frac{(p\epsilon)_\perp}{p_+} A,$$

where

$$A = \frac{q_2^2}{r_{2-}}(a^{(1)}, p + t_2) + \frac{q_1^2 q_2^2}{r_{2-}} + 2\frac{q_1^2 q_2^2}{r_{2-}}$$

$$+3\frac{q_1^2 q_{1+} r_{2-}}{r_{1-}} + \frac{r_2^2 q_{1+}^2}{q_{2+}} + 3\frac{q_1^2 q_{2+} r_{2-}}{r_{1-}} + \frac{q_2^2 r_1^2 q_{2+}}{q_{1+} r_{2-}} - (1 \leftrightarrow 2).$$

As we see each of the amplitudes $\mathcal{A}_i$, $i = 1, ..., 5$ contains both single poles in longitudinal momenta and double poles in longitudinal momenta of in-coming and out-going reggeons. Can they cancel in their sum together with terms obtained by permutation of reggeons? The answer is negative, since different amplitudes contain different denominators which moreover change with permutation of reggeons. These denominators depend on transverse momenta and so have different values. Therefore, at least at fixed transverse momenta the pole singularities contain different (and varying) coefficients in amplitudes $\mathcal{A}_i$, $i = 1, ..., 5$ and the amplitudes obtained from them by permutations of reggeons. So unlike the case of single projectile, in production amplitudes with two projectiles and targets poles in the longitudinal momenta remain uncanceled, which requires formulation of the way to do the longitudinal integrations. Integration in the principal value sense is an obvious choice.

7 Conclusions

We have derived the expression for the vertex RR→RRP describing gluon production in interaction of two in-coming and two out-going reggeons. The vertex can be used for calculations of inclusive cross-sections for gluon jet production in collision of a pair of projectile nucleons with a pair of target nucleons and also of the diffractive gluon jet production in deuteron-proton collisions. The vertex turns out to be quite complicated but amenable to further analytic and numerical calculations, which we postpone for future publications.

A few important properties of the obtained vertex have been demonstrated. The vertex is transversal in accordance with the gauge invariance. It vanishes when one of the longitudinal momentum goes to infinity, which allows to subsequently do integrations over longitudinal momenta in applications.

The vertex contains pole singularities at zero values of longitudinal momenta inherited from intermediate induced vertices in the framework of effective action. In the spirit of this framework one should consider these poles in the principal value sense. Note that in contrast to gluon production on several centers by a single projectile, where rescattering effects cancel these poles, in amplitudes containing the vertex RR→RRP, like shown in Fig. 1, there are no additional rescattering contributions, so that the mentioned pole singularities are preserved in the amplitudes and should be taken into account in longitudinal integrations.

Finally, again in contrast to the case of a single projectile [16, 17, 18], we find that the structure of the on-mass-shell vertex remains quite complicated and cannot be restored from the purely transverse picture, which is obtained by taking multiple cuts of the amplitude [19]. We believe that this is due to the fact that the amplitude possesses additional singularities, apart from the standard ones corresponding to physical intermediate gluons.

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9 Appendix. An alternative form of the amplitude $\mathcal{A}_5$

Here we present a more explicit form for the amplitude $\mathcal{A}_5$ in terms of coefficients in the two Lipatov vertices a_1, b_1, c_1 and a_2, b_2, c_2.

We rewrite the triple gluon vertex as

$$\Gamma_{\nu_1\mu,\nu_2}(t_1,p,t_2) = gf^{d_1c,d_2}\Big(\tau^{(1)}_{\nu_2}g_{\mu\nu_1} + \tau^{(2)}_{\nu_1}g_{\mu\nu_2} + \tau^{(3)}_{\mu}g_{\nu_1\nu_2}\Big),$$

where $\tau^{(1)} = -t_1 - p, \quad \tau^{(2)} = p + t_2, \quad \tau^{(3)} = t_1 - t_2$. This allows to write the final vertex as

$$\mathcal{A}_{5\mu} = g^3 C_5 \frac{1}{4(t_1^2 + i0)(t_2^2 + i0)}\Big(a_{1\mu}A^{(1)} + a_{2\mu}A^{(2)} + \tau^{(3)}_{\mu}A^{(3)} + n^+_{\mu}A^{(4)} + n^-_{\mu}A^{(5)}\Big), \qquad (59)$$

where

$$A^{(1)} = (a_2\tau^{(1)}) + b_2\tau^{(1)}_{+} + c_2\tau^{(1)}_{-}$$

$$A^{(2)} = (a_1\tau^{(2)}) + b_1\tau^{(2)}_{+} + c_1\tau^{(2)}_{-}$$

$$A^{(3)} = (a_2a_1) + b_1a_{2+} + c_1a_{2-} + +b_2a_{1+} + c_2a_{1-} + b_2c_1 + b_1c_2$$

$$A^{(4)} = b_1(a_2\tau^{(1)}) + b_2(a_1\tau^{(2)}) + b_1b_2(\tau^{(1)} + \tau^{(2)})_{+} + c_2b_1\tau^{(1)}_{-} + b_2c_1\tau^{(2)}_{-}$$

$$A^{(5)} = c_1(a_2\tau^{(1)}) + c_2(a_1\tau^{(2)}) + c_1c_2(\tau^{(1)} + \tau^{(2)})_{-} + c_2b_1\tau^{(2)}_{+} + b_2c_1\tau^{(1)}_{+}\ .$$

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